Investigation of heat transfer Prandtl number dependence by $u't'$ and $v't'$ solutions

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On the basis of second-order transport equations of velocity and temperature fluctuations, we present a numerical analysis of axial and radial turbulent heat flux for pipe flows with different Prandtl numbers. Distributions of different additives in the balance equations of axial and radial heat fluxes were analyzed under the effects of Re and *Pr*. Predicted parameter distributions were in a good agreement with experimental data of different authors over wide ranges of the operation parameters.

Keywords: convective heat transfer; turbulence; pipe flow; Prandtl number; turbulent heat flux; production; dissipation; numerical analysis

Introduction

The need for proper understanding of turbulent momentum and heat transfer challenges further investigations on different parameters of turbulence in wide ranges of the Reynolds and Prandtl numbers. We know a number of publications[1–3] that address different aspects of this problem. These analyses suggest that if the effects of Re and *Pr* on the heat transfer coefficient, temperature profile, intensity of temperature fluctuation in pipe flows of different fluids are examined deeply enough, Nu and ϑ^+ can be predicted by one of numerous relationships and will correlate with experimental results of different authors. But the available studies on turbulent heat flux densities $q_{ti} = -\rho c_p \overline{u_i' t'}$ are limited to flows of air ($Pr \simeq 0.7$), and only some of them refer to turbulent fluctuations of velocity and temperature in other fluids. Measurements of axial ($q_{tx} = -\rho c_p \overline{u't'}$) and radial ($q_{ty} = -\rho c_p \overline{v't'}$) heat fluxes for water in a pipe are described in Ref. 4; q_{tx} for ethyleneglycol ($Pr \simeq 53$) is described in Ref. 5, q_{ty} for mercury ($Pr \simeq 0.026$) is described in Ref. 6. The results of Refs. 4–6 suggest a close relation between the turbulent heat fluxes and *Pr*. This ought to be included in parameter predictions for turbulent heat transfer, since $\overline{u_i' t'}$ is a term in the equation of energy.

We think that any information on velocity and temperature fluctuations in pipe flows of differential fluids might be an important contribution to our knowledge of the turbulent transport process. One of the general approaches to the problem is simulation by using a second order closure approach for velocity and temperature fluctuations. But such equations are never closed because the initial equations of thermal and fluid dynamics are nonlinear.

We present not only closed form relations for axial and radial heat flux densities for turbulent heat transfer in flows of different *Pr*, but also radial distributions of separate terms of the balance of $\overline{u't'}_+$ and $\overline{v't'}_+$. Note that analytical[7] and experimental[8–10] studies on the distribution of terms in the equations of axial and radial heat fluxes cover only flows of gases ($Pr \simeq 0.7$).

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Preliminary data and relation

We consider the steady-state flow of a constant physical property fluid inside through a straight circular pipe under the assumption that temperature is a passive scalar. Then the balance equation of the second order moments of velocity and temperature fluctuations is

$$u_j \frac{\partial \overline{u_i' t'}}{\partial x_j} = -\frac{\partial}{\partial x_j}\left(\overline{u_i' t' u_j'} + \frac{1}{\rho}\overline{p't'}\right) + \frac{1}{\rho}\overline{p' \frac{\partial t'}{\partial x_j}} - \overline{u_j' t'}\frac{\partial U_i}{\partial x_j} - \overline{u_i' u_j'}\frac{\partial T}{\partial x_j}$$
$$+ \mathrm{k}\overline{u_i' \frac{\partial^2 t'}{\partial x_j \partial x_j}} + \nu \overline{t' \frac{\partial^2 u_i'}{\partial x_j \partial x_j}}, \qquad (i = 1, 2, 3) \qquad (1)$$

The left hand terms in Equation 1 stand for advection of $\overline{u_i' t'}$, the right hand ones stand for the transfer by turbulent diffusion and pressure fluctuation, the correlation between pressure fluctuation and the gradient of temperature fluctuation (pressure–strain scrambling), and the generation of the turbulent heat flux, respectively. The last two terms stand for molecular effects in $\overline{u_i' t'}$. But according to Ref. 11 such description is ill-posed, because it does not separate the effect of molecular viscosity and of conduction on molecular transfer and dissipation. From Ref. 12, two correct descriptions of the two components may be suggested

$$\nu \frac{\partial^2 \overline{u_i' t'}}{\partial x_j \partial x_j} - 2\nu \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial t'}{\partial x_j}} - (\nu - \mathrm{k})\overline{u_i' \frac{\partial^2 t'}{\partial x_j \partial x_j}} = A \qquad (2)$$

$$\mathrm{k} \frac{\partial^2 \overline{u_i' t'}}{\partial x_j \partial x_j} - 2\mathrm{k} \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial t'}{\partial x_j}} + (\nu - \mathrm{k})\overline{t' \frac{\partial^2 u_i'}{\partial x_j \partial x_j}} = A \qquad (3)$$

where A is the sum of the last two terms in Equation 1.

After adding the two expressions, we obtain

$$(\nu + \mathrm{k})\left[\frac{\partial^2 \overline{u_i' t'}}{\partial x_j \partial x_j} - 2\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial t'}{\partial x_j}}\right] + (\nu - \mathrm{k})\left[\overline{t' \frac{\partial^2 u'}{\partial x_j \partial x_j}} - \overline{u_i' \frac{\partial^2 t'}{\partial x_j \partial x_j}}\right] = 2A \qquad (4)$$

When $Pr = 1$ ($\nu \equiv \mathrm{k}$), the second term in the last left-hand relation is zero. We assume that for arbitrary *Pr* the values of the two square bracketed expressions are very close to one another and their difference may be neglected. Then the whole second left-hand term in Equation 4 may also be neglected as very small in comparison to the first term. Consequently the two last terms in Equation 1 may be rewritten in the following way

$$\mathrm{k}\overline{u_i'\frac{\partial^2 t'}{\partial x_j \partial x_j}}+\nu\overline{t'\frac{\partial^2 u_i'}{\partial x_j \partial x_j}}=(\nu+\mathrm{k})\left[\frac{\partial^2 \overline{u_i' t'}}{2\partial x_j \partial x_j}-\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial t'}{\partial x_j}}\right] \quad (5)$$

where the right-hand terms in Equation 5 stand for molecular transfer and for dissipation of $\overline{u_i't'}$, respectively.

We use approximations for the individual terms as in Ref. 7

$$\overline{u_i't'u_j'}+\frac{1}{\rho}\overline{p't'}=-\varepsilon_{t_i}\,\partial\overline{u_i't'}/\partial x_j \quad (6)$$

$$\frac{1}{\rho}\overline{p'\frac{\partial t'}{\partial x_i}}=-K_{t_i}\mathrm{E}^{0.5}\overline{u_i't'}/\iota_{u_it} \quad (7)$$

$$2\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial t'}{\partial x_j}}=C_t\overline{u_i't'}/\iota_{u_it}^2 \quad (8)$$

and assume a scale of correlation between velocity and temperature fluctuations proportional to the scale of turbulence in some functions of Pr

$$\iota_{u_it}=\psi_i(Pr)\cdot\iota \quad (9)$$

We show below that no explicit function of $\psi_i(Pr)$ is necessary, as it is absent in the final relation for axial and radial turbulent heat fluxes. We further assume that

$$\varepsilon_{t_i}=a_{t_i}\cdot\varepsilon_q, \qquad a_{t_i}=\text{constant} \quad (10)$$

For constant physical properties, the balanced equations for axial and radial heat fluxes have the following expression in dimensionless variables

$$U^+\frac{\partial\overline{u't'_+}}{\partial x^+}+V^+\frac{\partial\overline{u't'_+}}{\partial y^+}=\frac{1}{R}\frac{\partial}{\partial y^+}R[\psi(Pr)+a_{t_1}\varepsilon_q^+]\frac{\partial\overline{u't'_+}}{\partial y^+}$$
$$+\overline{v't'_+}\frac{\partial U^+}{\partial y^+}+\overline{u'v'_+}\frac{\partial\vartheta^+}{\partial y^+}-\overline{u't'_+}\left[\frac{C_{t_1}\psi(Pr)}{\iota_+^2\psi_1^2(Pr)}+\frac{K_{t_1}\mathrm{E}_+^{0.5}}{\iota_+\psi_1(Pr)}\right] \quad (11)$$

$$U^+\frac{\partial\overline{v't'_+}}{\partial x^+}+V^+\frac{\partial\overline{v't'_+}}{\partial y^+}=\frac{1}{R}\frac{\partial}{\partial y^+}R[\psi(Pr)+a_{t_2}\varepsilon_q^+]\frac{\partial\overline{v't'_+}}{\partial y^+}$$
$$+\overline{v_+'^2}\frac{\partial\vartheta^+}{\partial y^+}-\overline{v't'_+}\left[\frac{C_{t_2}\psi(Pr)}{\iota_+^2\psi_2^2(Pr)}+\frac{K_{t_2}\mathrm{E}_+^{0.5}}{\iota_+\psi_2(Pr)}\right] \quad (12)$$

Choice of numerical constants in approximated relations

To evaluate numerical values of the constants for dissipation and for diffusion in Equations 11 and 12, let us consider a fully-developed flow of steady-state heat transfer. In this situation we may neglect the advection terms in $\overline{u_i't'}$ so that Equations 11 and 12 become simple differential equations.

In the logarithmic law region that is far-fetched from either the wall and the core, diffusion in $\overline{u_i't'}$ may be neglected and Equation 11 becomes

$$\overline{v't'_+}\frac{dU^+}{dy^+}+\overline{u'v'_+}\frac{d\vartheta^+}{dy^+}=\overline{u't'_+}\left[\frac{C_{t_1}\psi(Pr)}{\iota_+^2\psi_1^2(Pr)}+\frac{K_{t_1}\mathrm{E}_+^{0.5}}{\iota_+\psi_1(Pr)}\right] \quad (13)$$

This layer follows a number of specific functions $dU^+/dy^+=1/\kappa y^+$, $d\vartheta^+/dy^+=Pr_t/\kappa y^+$, $\mathrm{E}_+=$const., $\overline{u'v'_+}/\mathrm{E}_+=$const., $Pr_t=$ const., $\overline{u't'_+}/\overline{v't'_+}=$const. and $\iota_+\sim y^+$. Then it is easy to prove

Notation

$a_{t_1}, a_{t_2}, C_{t_1}, C_{t_2}$	Empirical constants
E	Kinematic energy of turbulence
E_+	E/u_*^2
Gr_A	Grashof number based on axial temperature gradient $\equiv\rho^2\beta g(d\overline{T}/dx)d^4/\mu^2$
k	Coefficient of heat conductivity
K_{t_1}, K_{t_2}	Empirical constants
ι	Scale of turbulence
ι_+	$\iota u_*/\nu$
p'	Pressure fluctuation
Pr	Prandtl number $\equiv\nu/k$
Pr_t	Turbulent Prandtl number
r	Current radius
r_0	Radius of pipe
R	r/r_0
Re	Reynolds number $\equiv 2r_0U/\nu$
T	Temperature
t'	Temperature fluctuation
U	Average axial velocity
U^+	U/u_*
u'	Axial velocity fluctuation
u_*	Dynamic velocity
$\overline{u'v'}$	Turbulent shear stress
$\overline{u'v'_+}$	$-\overline{u'v'}/u_*^2$
$\overline{u't'}$	Axial turbulent heat transfer
$\overline{u't'_+}$	$\overline{u't'}/u_*$
V	Average radial velocity
V^+	V/u_*
v'	Radial velocity fluctuation
$\overline{v'^2}$	Intensity of radial velocity fluctuation
$\overline{v_+'^2}$	$\overline{v'^2}/u_*^2$
$\overline{v't'}$	Radial turbulent heat transfer
$\overline{v't'_+}$	$-\overline{v't'}/u_*\vartheta_*$
x	Longitudinal coordination axis
x^+	xu_*/ν
y	Distance from the wall $\equiv r_0-r$
y^+	yu_*/ν
ε_q	Coefficient of turbulent heat conduction $\equiv-\overline{v't'}/(dT/dy)$
ε_q^+	ε_q/ν
ε_t	Coefficient of turbulent heat transfer
ϑ_*	Scale of temperature
ϑ^+	$(T-T_w)/\vartheta_*$
ν	Kinematic viscosity coefficient
ρ	Density
ψ	0.5(1+1/Pr)
ψ_1, ψ_2	Empirical functions
τ_w	Wall shear stress
τ	Time

Subscripts

t	Turbulent
w	Wall
x	Axial
y	Radial

a ratio of $1:1/y^+:1(y^+ \gg 1)$ among the orders of the left-hand terms, and the first and the second right-hand terms in the last relation. So in this layer the contribution of the dissipation term in $\overline{u't'_+}$ is neglectful as compared to the terms for generation and for pressure–strain scrambling, so that Equation 11 may be written

$$\overline{v't'_+}\frac{dU^+}{dy^+}+\overline{u'v'_+}\frac{d\vartheta^+}{dy^+}=K_{t_1}\frac{\overline{u't'_+}\cdot \mathrm{E}_+^{0.5}}{\imath_+\cdot\psi_1(Pr)} \tag{14}$$

But with

$$Pr_t=\frac{\overline{u'v'_+}}{\overline{v't'_+}}\frac{d\vartheta^+/dy^+}{dU^+/dy^+} \tag{15}$$

we find from Equation 14

$$K_{t_1}=\frac{(1+Pr_t)\psi_1(Pr)\overline{v't'_+}}{\overline{u't'_+}\cdot\mathrm{E}_+^{0.5}}\imath_+\frac{dU^+}{dy^+} \tag{16}$$

We have for the scale of turbulence from the relation of Prandtl

$$\overline{u'v'_+}=\imath_+^2(dU^+/dy^+)^2 \tag{17}$$

and therefore find the following relation for the limiting K_{t_1}

$$K_{t_1}=(1+Pr_t)(\overline{u'v'_+}/\mathrm{E}_+)^{0.5}(\overline{v't'_+}/\overline{u't'_+})\cdot\psi_1(Pr) \tag{18}$$

Experimental data for different flow modes[13–15] suggest ratio values of turbulent shear stress and energy of turbulence close to 0.3. We assume here that $\overline{u'v'_+}/\mathrm{E}_+=0.26$, as in Ref. 7. Experimental data of axial and normal turbulent heat fluxes[16–18] indicate nearly constant values of $\overline{u't'_+}/\overline{v't'_+}$ about 1.7 over most of the pipe cross-section (in the range of y/r_0 from 0.1 to 0.9). We assume for this flow regime that $Pr_t=0.87$ as in Refs 7, 18 and find K_{t_1}

$$K_{t_1}=0.56\psi_1(Pr) \tag{19}$$

To find the value of C_{t_1}, let us consider the balance equation of turbulent heat flux in the vicinity of the wall. By maintaining in Equation 11 the terms for generation and dissipation, we find

$$\overline{v't'_+}\frac{dU^+}{dy^+}+\overline{u'v'_+}\frac{d\vartheta^+}{dy^+}=C_{t_1}\frac{\overline{u't'_+}\cdot\psi(Pr)}{\imath_+^2\cdot\psi_1^2(Pr)} \tag{20}$$

But because of Equation 15 we find from Equation 20

$$C_{t_1}=(1+Pr_t)\frac{\overline{v't'_+}}{\overline{u't'_+}}\frac{\psi_1^2(Pr)}{\psi(Pr)}\imath_+^2\frac{dU^+}{dy^+} \tag{21}$$

With similar assumptions we find from the balance equation of turbulent shear stress given in Ref. 7

$$\imath_+^2\, dU^+/dy^+=C_2\overline{u'v'_+}/\overline{v'^2_+},\qquad C_2=22 \tag{22}$$

The following relation for the near wall region was found experimentally in Ref. 18

$$\frac{\overline{u't'_+}}{\overline{v't'_+}}=B_1\frac{\overline{u'v'_+}}{\overline{v'^2_+}}\frac{(1+Pr_t)}{Pr_t},\qquad B_1=1.3 \tag{23}$$

To find C_{t_1} we introduce Equation 22 and 23 into Equation 21

$$C_{t_1}=\frac{C_2 Pr_t}{B_1}\frac{\psi_1^2(Pr)}{\psi(Pr)}\simeq 14.72\frac{\psi_1^2(Pr)}{\psi(Pr)} \tag{24}$$

A direct evaluation of constant a_{t_1} from experimental data involves their differentiation, which means a high error. Therefore, the value of constant a_{t_1} was found numerically by comparing predicted distributions of the diffusion term in Equation 11 to the corresponding experimental results.[10] The best agreement was found with $a_{t_1}=0.15$.

Numerical values of constants in the relation for the turbulent radial heat flux[12] are evaluated in a similar way. A detailed description for a gas flow in a pipe is given in Ref. 7. By similar considerations for arbitrary Pr, descriptions of C_{t_2}, K_{t_2}, and a_{t_2} can be found:

$$C_{t_2}=18.9\psi_2^2(Pr)/\psi(Pr),\qquad K_{t_2}=1.01\psi_2(Pr),\qquad a_{t_2}=0.15 \tag{25}$$

Initial relations

We introduce a_{t_1}, a_{t_2}, C_{t_1}, C_{t_2}, K_{t_1}, and K_{t_2} in Equations 11 and 12 to formulate initial relations for the balance of axial and radial heat fluxes

$$U^+\frac{\partial\overline{u't'_+}}{\partial x^+}+V^+\frac{\partial\overline{u't'_+}}{\partial y^+}=\frac{1}{R}\frac{\partial}{\partial y^+}R[\psi(Pr)+0.15\varepsilon_q^+]\frac{\partial\overline{u't'_+}}{\partial y^+}$$
$$+\overline{v't'_+}\frac{\partial U^+}{\partial y^+}+\overline{u'v'_+}\frac{\partial\vartheta^+}{\partial y^+}-\overline{u't'_+}(14.72/\imath_+^2+0.56\mathrm{E}_+^{0.5}/\imath_+) \tag{26}$$

$$U^+\frac{\partial\overline{v't'_+}}{\partial x^+}+V^+\frac{\partial\overline{v't'_+}}{\partial y^+}=\frac{1}{R}\frac{\partial}{\partial y^+}R[\psi(Pr)+0.15\varepsilon_q^+]\frac{\partial\overline{v't'_+}}{\partial y^+}$$
$$+\overline{v'^2_+}\frac{\partial\vartheta^+}{\partial y^+}-\overline{v't'_+}(18.9/\imath_+^2+1.01\mathrm{E}_+^{0.5}/\imath_+) \tag{27}$$

Equations 26 and 27 were solved simultaneously with those for ϑ^+ and ε_q^+ [7]

$$U^+\frac{\partial\vartheta^+}{\partial x^+}+V^+\frac{\partial\vartheta^+}{\partial y^+}=\frac{1}{R}\frac{\partial}{\partial y}R\left(\frac{1}{Pr}\frac{\partial\vartheta^+}{\partial y^+}-\overline{v't'_+}\right) \tag{28}$$

$$\varepsilon_q^+=\overline{v't'_+}/(d\vartheta^+/dy^+) \tag{29}$$

with the boundary conditions

$$\vartheta^+=\overline{u't'_+}=\overline{v't'_+}=0\qquad\text{when } y^+=0 \tag{30}$$

$$d\overline{u't'_+}/dy^+=\overline{v't'_+}=0\qquad\text{when } y^+=y_0^+ \tag{31}$$

The assumed values of U^+, $\overline{u'v'_+}$, $\overline{v'^2_+}$, E_+ and l_+ were taken from Ref. 7.

Numerical techniques

The initial set of relations, Equations 26–29; with boundary conditions described by Equations 31 and 32 was solved by well tested finite-difference method for the boundary layer equations,[19] combined with transient technique[20] for the solution of the transport equations of $\overline{u't'_+}$ and $\overline{v't'_+}$ by introducing local derivatives of time-dependent variables. The transient technique was employed because a satisfactory convergence cannot be achieved by numerical iteration, even with specific steps for the improvement of its convergence rate.[20]

A finite-difference approach was applied to the transient differential equations, corresponding to Equations 26 and 27. This was done in an implicit two-layer six-point scheme.[19] The resulting set of finite-difference equations was solved by the method of elimination followed by an iterative procedure.[21]

Iteration was stopped whenever the condition of

$$|f^{s+1}-f^s|\leqslant\varepsilon_1\cdot f^s \tag{32}$$

was satisfied. Here, s is the number of iteration and f is one of the sought variables $\overline{u't'_+}$ or $\overline{v't'_+}$. Steady-state values are achieved when condition

$$|f^{j+1}-f^j|\leqslant\varepsilon_2\cdot\Delta\tau \tag{33}$$

is satisfied for three consequent values of j, which is the time-dependent number of layers. The conditions must be valid throughout the whole cross-section of the pipe. The prediction

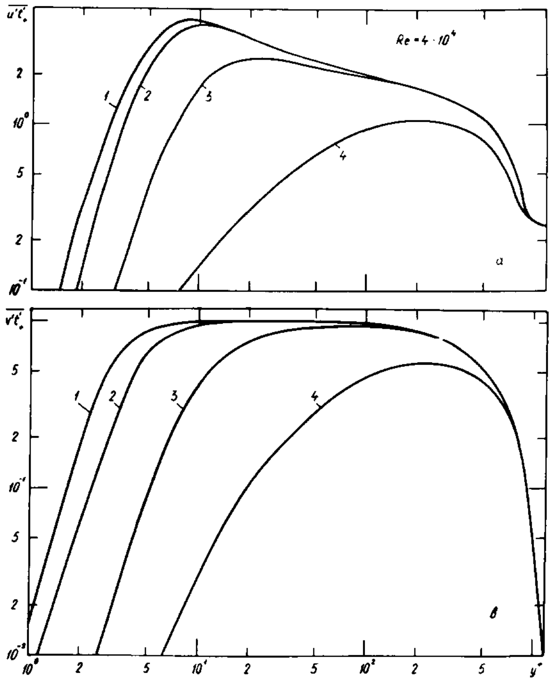

Figure 1 Distribution of axial (a) and radial (b) turbulent fluxes in the cross section. *Pr*: 1—53, 2—14.3, 3—0.7, 4—0.026

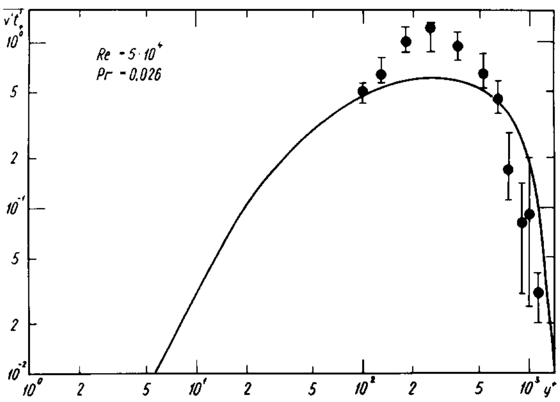

Figure 2 Comparison of predicted and experimental radial turbulent heat fluxes for mercury[6]

was preceded by a test run, when subsequent values $\varepsilon_1 = \varepsilon_2 = 10^{-4}$ of the iteration parameter and the transient criterion were found.

Depthwise iteration in the boundary layer was performed with a variable step, which was doubled in arbitrary locations. The number of the double-step points was chosen in such a way that, for a constant number of radial points equal to 64, the near-wall step never exceeded $y^+ = 0.05$. In this way, the considered ranges of operation parameters always included a certain number of points in the viscous sublayer.

Automated choice of step along x^+ was performed with the account of the number of iteration steps in the preceding cross-section. Whenever the number of iteration steps K_n' in the n-th cross section exceeded the pregiven number K_n, the step in the $(n+1)$-th cross section was $\Delta x^{+(n+1)} = \Delta x^{+(n)}/2$. Otherwise $\Delta x^{+(n+1)} = 1.1\Delta x^{+(n)}$. Preliminary calculations suggested that, for turbulent flows, the results are loosely related to the initial step $\Delta(x_{in}^+)$ along x^+, and with two values of Δx^+ different by a factor of five (say $\Delta x_{in}^+ = 0.02$ and $\Delta x_{in}^+ = 0.1$), a deviation was only noted near the entrance to the heated section.

Numerical results

A distribution analysis of $\overline{u't'_+}$ shows (Figure 1(a)) that with high *Pr*, the maximum in the distribution is increased and shifted towards the wall. Thus, an increase in *Pr* expands the region where axial turbulent heat fluxes dominate in the longitudinal heat transfer. The different molecular temperature diffusivity of the liquids has a very strong influence on the radial distribution of turbulent heat fluxes, especially near the wall (Figure 1(b)) and the value of $\overline{v't'_+}$ increases with *Pr*.

Predicted distributions of axial and radial turbulent heat fluxes are in a satisfactory qualitative agreement with experimental data for liquid metals (Figure 2),[6] air (Figure 3),[14,22] water (Figure 4)[4] and ethylene glycol (Figure 5).[5] Note also a satisfactory quantitative agreement with the most reliable results for water and air. One of the evident causes for the significant numerical deviations between the analysis and the experiment is the assumption of passive temperature in flows of liquid metals and ethylene glycol.[5,6] The experiments involved rather high heat fluxes, so that the assumption was not true for the parameters of forced turbulent transfer because of a free-convection effect. As indicated in Ref. 23, experimental data in Ref. 6 were accumulated with $Gr_A/\mathrm{Re}^2 = 4.15 \cdot 10^{-4}$. To our regret, we could not find any data on the effects of buoyancy in forced momentum and heat transfer for the experimental conditions of Ref. 5. Our evaluations suggest $Gr_A/\mathrm{Re}^2 \simeq 1.69 \cdot 10^{-5}$. Ref. 24 notes an influence of buoyancy on the heat transfer as early as at $Gr_A/\mathrm{Re}^2 > 2 \cdot 10^{-4}$. Note from Ref. 12 that the effect of buoyancy becomes evident in the turbulent transport in the first rate, only later in average velocity and temperature profiles, and in the last rate in integral parameters, such as hydraulic drag and heat transfer. Therefore, the deviation between the prediction and the experiment in Ref. 6 may be reliably ascribed to free convection.

The deviation between the prediction and the experiment in turbulent axial heat flux distributions for ethylene glycol ($Pr = 53$) in Ref. 5 must be related to the assumption of the negligibly small second left-hand term in Equation 4 for *Pr* very different from one. This assumption may well be subject

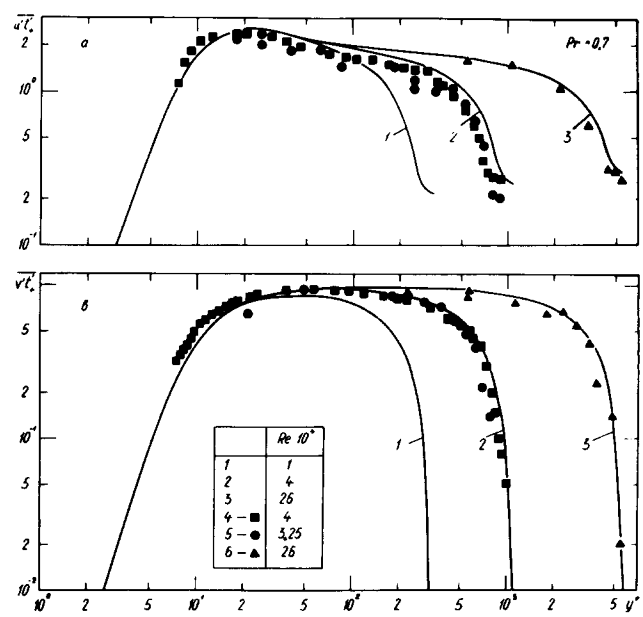

Figure 3 Comparison of predicted and experimental axial (a) and radial (b) turbulent heat fluxes for air. 4—Ref. 22; 5, 6—Ref. 16. Error rates: Ref. 16—5–8%, Ref. 22—5.8%

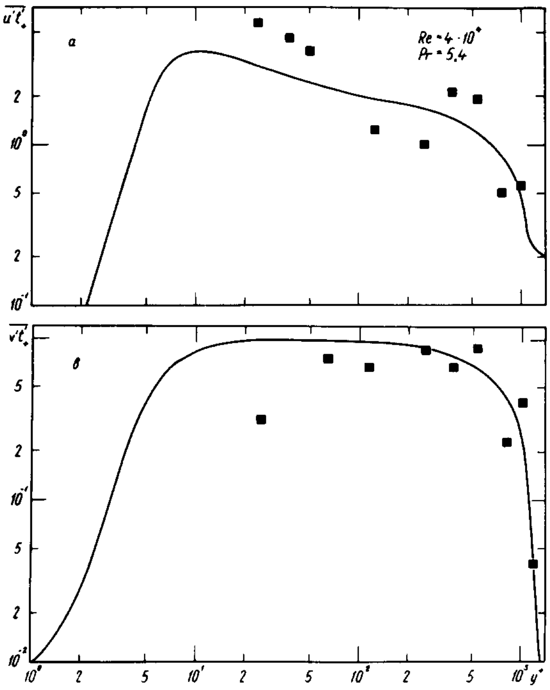

Figure 4 Comparison of predicted axial turbulent heat fluxes for water with experiment[4]

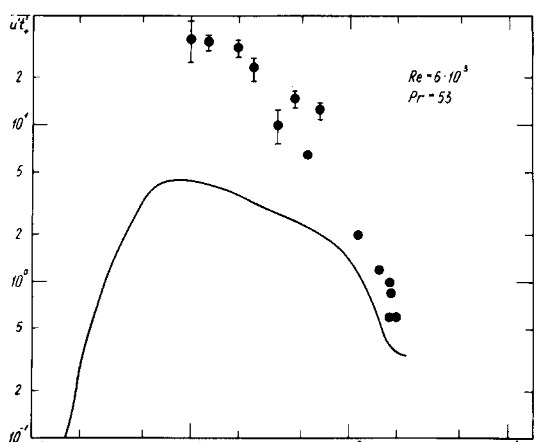

Figure 5 Comparison of predicted axial turbulent heat fluxes for ethylene glycol with experiment[5]

to serious reconsideration, when reliable experimental data becomes available.

Near-wall distributions of $\overline{u't'_+} = f_1(y^+)$ and $\overline{v't'_+} = f_2(y^+)$ do not depend on Re, except after the maxima of $\overline{u't'_+}$ and $\overline{v't'_+}$ (Figure 3).

One more important factor, especially in predictions of forced flow and heat transfer including buoyancy, is the ratio $\overline{u't'_+}/\overline{v't'_+}$. Thus far, its measurements cover only gases,[16–18] ($Pr = 0.7$). This is the reason why different models of turbulence assume $\overline{u't'_+}/\overline{v't'_+}$ independent of Pr. In one of the studies[25] the following relation was suggested

$$\overline{u't'_+}/\overline{v't'_+} = 280\exp(0.632\cdot10^{-4}\mathrm{Re})\exp(-20y/r_0) + 1.6 \quad (34)$$

Our prediction (Figure 6, curves 1–4) suggests a complex relation between $\overline{u't'_+}/\overline{v't'_+}$ and Pr, and especially near the wall. Here $\overline{u't'_+}/\overline{v't'_+}$ decreases with the growth of Pr, and becomes independent of Pr at $y^+ = 20$. Predicted distributions of $\overline{u't'_+}/\overline{v't'_+}$ in the cross-section area are independent of Re (Figure 6(b)). This observation is supported by experiments of different authors.[16–18]

Respective distributions of separate terms in the equations of $\overline{u't'_+}$ and $\overline{v't'_+}$ for different Pr are shown in Figures 7 and 8.

Terms $\overline{v't'_+}\, d\vartheta^+/dy^+ + \overline{u'v'_+}\, d\vartheta^+/dy^+$ and $\overline{v_+'^2}\, d\vartheta^+/dy^+$ stand for the generation of turbulent axial and radial heat fluxes caused by deformations of average velocity and temperature profiles. Generation of $\overline{u't'_+}$ and $\overline{v't'_+}$ is mainly determined by deformations of average temperature profiles and occurs in the regions of a linear behavior of ϑ^+ and in the buffer zone (that

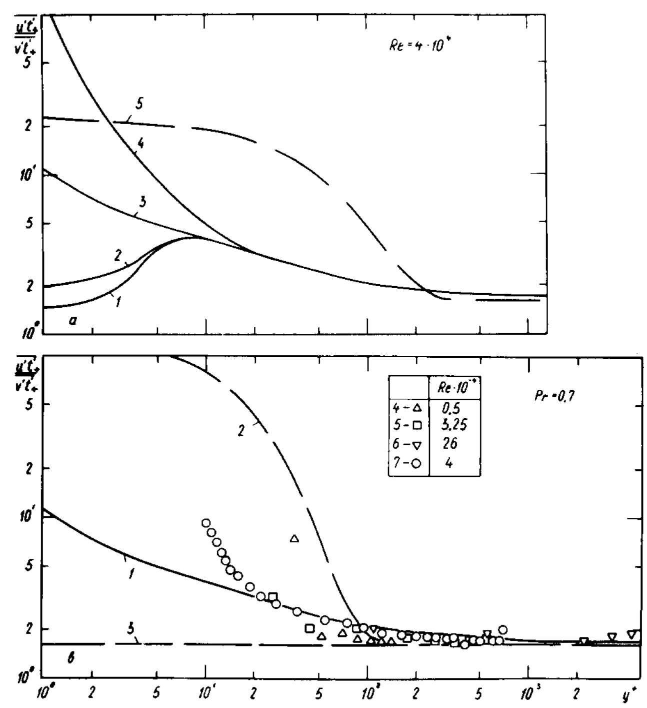

Figure 6 Comparison of predicted relations of $\overline{u't'_+}/\overline{v't'_+}$ to Pr: (a) with experiments; (b) *a*: 1—53, 2—14.3, 3—0.7, 4—0.026, 5—after Equation 34; *b*: curves: 1—present study, $\mathrm{Re} = 10^4$ to $2.6\cdot10^5$, 2, 3—after Equation 34, $\mathrm{Re} = 10^4$ and $2.6\cdot10^4$, respectively; points-experiment: 4—Ref. 17, 5, 6—Ref. 16, 7—Ref. 18

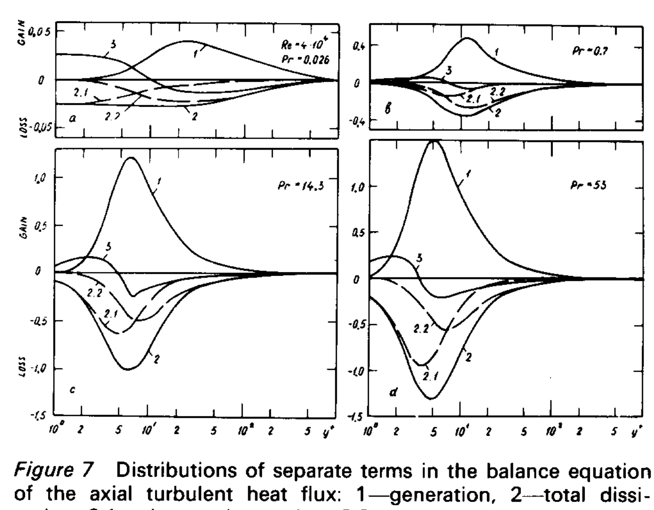

Figure 7 Distributions of separate terms in the balance equation of the axial turbulent heat flux: 1—generation, 2—total dissipation, 2.1—viscous destruction, 2.2—pressure-strain scrambling, 3—diffusion

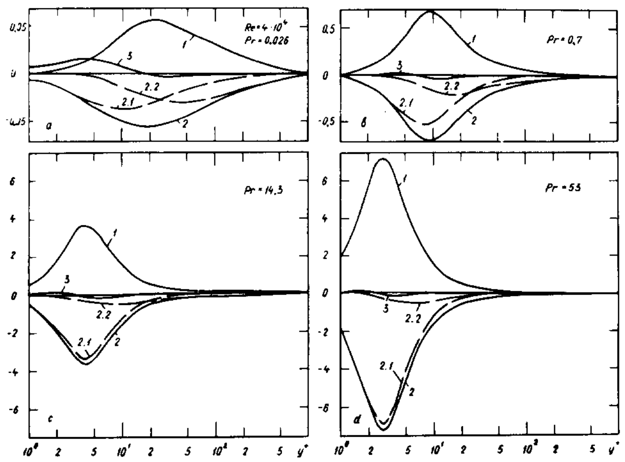

Figure 8 Distributions of separator terms in the balance equation of the radial turbulent heat flux. Notations as in Figure 7

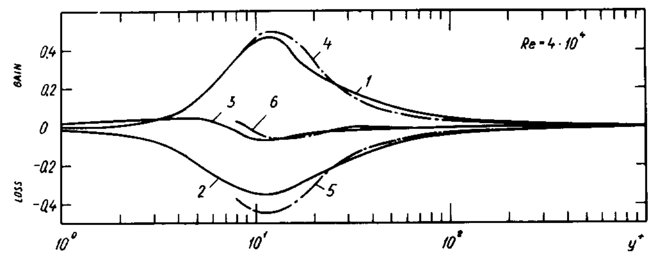

Figure 9 Comparison of predicted (solid lines) terms in the balance equation of the axial turbulent heat flux with experimental data for air.[10] 1, 4—generation, 2, 5—dissipation, 3, 6—diffusion

is, between the regions of linear and logarithmic distributions of average temperature) of the thermal boundary layer. Here the gradients $d\vartheta^+/dy^+$ undergo sharpest changes. In the buffer zone of thermal boundary layer average temperature gradients are most pronounced, and this is the region where generations of $\overline{u't'_+}$ and $\overline{v't'_+}$ achieve their maximum values. On the account of diffusion of $\overline{u't'_+}$ and $\overline{v't'_+}$, the thermal energy transport goes from the region of maximum generation to the whole buffer zone and linear zone of the thermal boundary layer. Actually, equilibrium between generation and dissipation of turbulent axial and radial heat fluxes is in the whole cross-section area, except the near-wall layer. By dissipation in the balance equation of $\overline{u't'_+}$ and $\overline{v't'_+}$ we mean the sum of terms for viscous destruction and pressure–strain scrambling. Near the wall the predicted distributions of separate terms in equations for $\overline{u't'_+}$ and $\overline{v't'_+}$ are not related to Re. In the rest, part of the flow of the values, but not the behavior of these terms, is related to Re. The influence of *Pr* on the distribution of separate terms in the transport equations for the turbulent axial and radial heat fluxes is most pronounced in the region of largest deformations of average temperature. The values, shapes and locations of extreme points of generation, dissipation and diffusion are all related to *Pr*. With an increase of *Pr*, the ranges of sharpest changes in the component of the balance of $\overline{u't'_+}$ and $\overline{v't'_+}$ are reduced, because of the reduction of the regions of sharp average-temperature gradients. This is accompanied by an increase of relative values of viscous destructions (Figure 8, curves 2.1) in the total dissipation of $\overline{u't'_+}$ and $\overline{v't'_+}$ (Figure 8, curves 2), as compared to pressure strain scrambling (Figure 8, curves 2.2). The effect is especially pronounced in the viscous destruction and pressure–strain scrambling terms in the transfer equation of $\overline{v't'_+}$. Further from the wall, similar to Re, *Pr* is a determining factor for the amounts of the terms in the equations for $\overline{u't'_+}$ and $\overline{v't'_+}$, but not for their behavior in the cross-section. Note that at low *Pr* the contribution of the terms for diffusion of $\overline{u't'_+}$ and $\overline{v't'_+}$ is significant over nearly the whole stream because of the similar levels of molecular and turbulent transfer far from the wall. The insignificant level of diffusion as compared to that of dissipation and generation in the balance of $\overline{v't'_+}$ far from the wall is the reason of the validity of algebraic models of turbulent heat transfer at high and moderate *Pr*. With neglected terms of diffusion, Equation 22 is easily reduced to an algebraic one.

The equations of balance of the second moments are closed with the help of several hypothetical approximations, and, as shown in Ref. 26, comparisons of such predictions with experiments must be considered solely as a necessary condition for the reliability of the initial equations. Only separate validations of each hypothesis constitutes a sufficient condition for their reliability. Hypotheses treated in Ref. 26 serve for the approximation of unknown terms of the balanced equations of the second moments. Therefore only the agreement between predicted and experimental distributions of the terms in the balanced equations of axial and radial heat fluxes may be a sufficient condition for their validity. The experiments with pipe flows cover only gases,[10] and predictions are in a good agreement with the experiments (Figure 9).

Conclusions

Closed equations suggested for axial and radial heat fluxes for pipe flows with different *Pr* yield the values of turbulent heat transfer and the distributions of the separate terms in the balance equations for $\overline{u't'_+}$ and $\overline{v't'_+}$. They are close to experimental results of different authors in wide ranges of Re and *Pr*. We conclude:

(a) The ratio $\overline{u't'_+}/\overline{v't'_+}$ decreases with a growth of *Pr* near the wall, but is independent of *Pr* at $y^+ > 20$.
(b) The ratio $\overline{u't'_+}/\overline{v't'_+}$ is independent of Re over the whole pipe cross sectional area.
(c) The influences of Re and *Pr* on the distributions of separate terms in the transport relations of turbulent axial and radial heat fluxes is most pronounced in the region of largest deformations of average temperature. The magnitudes, shapes and locations of the extreme points of generation, dissipation and diffusion of $\overline{u't'_+}$ and $\overline{v't'_+}$ are influenced by both Re and *Pr*.
(d) At large and moderate *Pr*, the contributions by diffusion in the balance of $\overline{v't'_+}$ constitutes several percent of the contribution by dissipation-generation. This provides an explanation for the good performance of algebraic models of turbulent heat exchange for such *Pr*.

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